International Journal of Engineering Sciences 2024 16(1) 41 - 53

INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES

Numerical and Dynamic Analysis of Dual-Arm Robotic Manipulation Using Newton-Euler Formulation

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(Received 22 March 2024; Accepted 28 September 2024)

https://doi.org/10.36224/ijes.160105

Abstract
This paper presents a comprehensive dynamic modeling and simulation-based analysis of a dual-arm robotic system, each arm consisting of seven degrees of freedom (DOF). The study employs the Newton-Euler recursive formulation to perform inverse dynamic analysis, enabling the calculation of joint torques required for executing coordinated motion. A 100 mm cubic object is considered as a shared payload between the arms to simulate cooperative manipulation. Kinematic parameters such as joint angles, velocities, and accelerations are generated using smooth sinusoidal trajectories to ensure realistic and continuous motion. The resulting torque profiles are analyzed under dynamic loading conditions, including the influence of the object's mass. Graphical results for both arms confirm that the system maintains smooth and synchronized motion with torque values well within feasible limits. The analysis highlights the effectiveness of the Newton-Euler approach for accurately modeling complex dual-arm robotic systems and provides valuable insights for real-time control and cooperative task execution

Keywords: dynamic modeling, robotic system, inverse dynamics, Newton-Euler

1. Introduction

Dual-arm robotic systems are increasingly employed in complex manipulation tasks that require coordinated bimanual control, such as object assembly, surgical assistance, and collaborative manufacturing. These systems emulate human-like behavior, offering enhanced flexibility, redundancy,and the capability to manipulate large or deformable objects with precision [1]. The coordination of two manipulators introduces additional challenges in motion planning, control, and dynamic modeling,particularly when interacting with a common payload.Dynamic analysis plays a critical role in ensuring stability, control accuracy, and safe operation of robotic systems. It enables the prediction of joint forces and torques, facilitating optimal actuator selection, control design, and real-time execution of complex trajectories [2]. Among the widely adopted methods for dynamic modeling, the Newton-Euler recursive formulation is recognized for its computational efficiency and ability to handle serial linkages in real-time scenarios [3]. This method computes joint torques by performing a forward pass to propagate velocities and accelerations, followed by a backward pass to calculate forces and moments.This study presents a comprehensive dynamic simulation of a 7-DOF dual-arm robotic system executing a cooperative manipulation task

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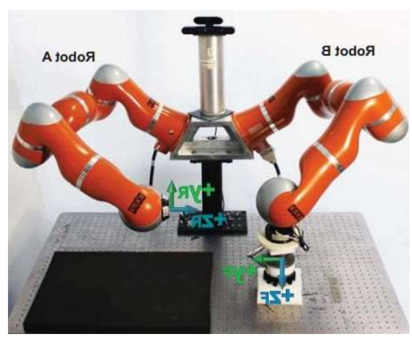
ISSN 0976 – 6693.

involving a 100 mm cubic object. Kinematic profiles including joint angles, velocities, and accelerations are generated using smooth sinusoidal trajectories to ensure continuity and realism. The Newton-Euler method is employed to calculate the joint torques for each arm, taking into account randomized inertial parameters and external object dynamics. Graphical results are used to evaluate the performance of each joint in terms of motion smoothness and torque limits. The analysis provides valuable insights into the feasibility of dual-arm manipulation and serves as a reference for developing real-time control strategies for collaborative robots [4].The recent advancements in robotics have led to the transition from single-arm industrial manipulators to more sophisticated dual-arm humanoid systems capable of performing complex, human-like tasks.While traditional manipulators face limitations such as low payload capacity and limited operational flexibility, dual-arm robots offer enhanced dexterity, cooperative handling of rigid objects, and improved performance in unstructured environments. These systems are increasingly being explored for applications across manufacturing, healthcare, and service domains, where human-robot collaboration is essential. [5-8]


(a) Apollo Humanoid Robot [5] (b) KIT Dual Arm System [7]

(c) CROM Dual Arm robot [8] (d) KUKA Dual Arm Setup [9]

Fig. 1. Different types of dual arm system for various applications

2. Literature review

The development of robotic systems that can emulate human-like dexterity and perform sophisticated manipulation tasks has been a central objective in robotics research for decades [8]. A key component in realizing this capability is the formulation of accurate dynamic models for dual-arm manipulation systems. These models mathematically describe the interaction between robotic arms, manipulated objects, and the surrounding environment, providing the foundation for implementing control strategies that ensure coordinated and efficient movements. Traditionally, dynamic modeling approaches have considered joints as rigid and employed the Lagrangian method to construct individual models for each manipulator. Simultaneously, object dynamics are typically represented using classical force and torque balance principles, as outlined in references [10,11]. Recent advancements include the adoption of the Udwadia-Kalaba formulation, offering alternative solutions for modeling constrained mechanical systems. However, the Lagrangian framework remains prevalent in scenarios involving unconstrained or open-ended dynamics, often increasing model complexity [12].

Lv et al. proposed a discrete elastic rod-based model for dynamic control of deformable linear objects using ABB Yumi, enhancing shape accuracy and control [13]. Guo et al. developed a nonlinear model predictive control framework for free-floating dual-arm space robots, improving trajectory tracking and base stability [14]. Rader et al. introduced the KIT Dual Arm System with 8-DOF per arm and inner shoulder joints, achieving greater dexterity and human-like motion, demonstrating a balanced approach between industrial robustness and humanoid flexibility [7].

Some researchers have extended dynamic analysis to cooperative manipulators with flexible links, employing hybrid modeling approaches that combine the recursive Gibbs-Appell formulation for robotic arms with the Newton-Euler method for object dynamics [15]. These models offer deeper insights into the internal forces and coupling effects during cooperative manipulation. Moreover, humanoid dual-arm systems require detailed mathematical formulations encompassing both kinematic and dynamic aspects, along with interaction forces between the limbs and objects [16]. These predictive models help simulate real-world behavior by integrating mechanical dynamics, control theory, and numerical algorithms, ultimately enabling robots to execute tasks with higher efficiency and adaptability.

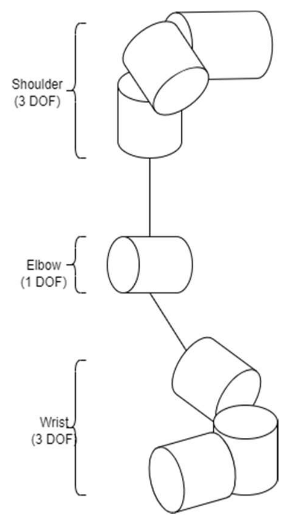


Fig. 2. 7 DOF Arm representation

In this work, a dynamic model of a 7-DOF dual-arm robotic system was developed and implemented in MATLAB using the Newton-Euler recursive formulation. The model incorporates both individual arm dynamics and cooperative manipulation of a 100 mm cubic object. Trajectory planning was performed for smooth motion generation, and joint torques were computed for both arms under object-free and object-carrying conditions. Simulation results showed that for the left arm, joint angles varied between 0.5 radians, velocities reached up to ±2.5 rad/s, and accelerations peaked at ±15 rad/s². Similarly, the right arm demonstrated joint angles in the range of 0.5 radians, velocities up to ±2.0 rad/s, and accelerations up to ±12 rad/s². Torque analysis revealed that certain joints experienced significant load variation when the object was introduced, highlighting the importance of dynamic load consideration in cooperative tasks.

3. Jacobian Derivation via Velocity Propagation

3.1 Fundamental of Analytical Jacobian in Robot Kinematics

The analytical Jacobian also relates joint velocities to end-effector velocities but is derived through differentiation of the forward kinematics equations. It considers both the geometric properties of the robot and the joint space dynamics, including joint limits and velocity limits. The analytical Jacobian is more comprehensive as it incorporates joint limits and velocity constraints, making it suitable for tasks like inverse kinematics and robot control, where accurate joint motions are crucial. The analytical Jacobian matrix $Janalytical$ is typically represented as follows:

$$J_{\text{analytical}} = \begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} & \cdots & \frac{\partial f_1}{\partial u_p} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} & \cdots & \frac{\partial f_2}{\partial u_p} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial u_1} & \frac{\partial f_n}{\partial u_2} & \cdots & \frac{\partial f_n}{\partial u_p} \end{bmatrix} \tag{1}$$

where, fk represents the k^{th} component of the end-effector position or orientation (e.g., *x, y, z* or Euler angles)

uj represents the j^{th} joint variable
n is the number of end-effector components (e.g. m=6 for a 6-DOF robotic arm).
r is the number of joint variables.

Each element of the analytical Jacobian matrix represents the partial derivative of the corresponding EE component w.r.t each joint variable. The analytical Jacobian provides a precise representation of the relationship between joint velocities and end-effector velocities. It is particularly useful for robotic systems with well-defined kinematic equations, as it allows for efficient computation without relying on numerical approximations.

3.2 Fundamental of Geometric Jacobian in Robot Kinematics

The geometric Jacobian is an essential concept in robotics and mechanical systems that describes how joint or actuator motions influence the velocity and orientation of the end-effector (EE) or any specific point of interest on a robot. Essentially, it defines the kinematic relationship between joint velocities and the resulting motion of the robot's links or end-effector, based purely on the robot's geometric structure and configuration.

From a mathematical perspective, the geometric Jacobian establishes a connection between the velocities of the joints and the velocity of the chosen frame or point on the manipulator. This relationship is expressed
as:

$$\dot{x} = J(q)\,\dot{q} \tag{2}$$

Here:

- $\dot{x}$ denotes the velocity of the reference point or frame,
- $J(q)$ is the geometric Jacobian matrix, which varies with the robot's configuration q,
- $\dot{q}$ represents the vector of joint velocities.

The geometric Jacobian matrix is typically represented as follows:

$$J \quad Jr$$

where,
$geom = [J_t]$

Jr corresponds to the rotational part of the Jacobian, representing the angular velocities

- Jt corresponds to the rotational part of the Jacobian, representing the linear velocity

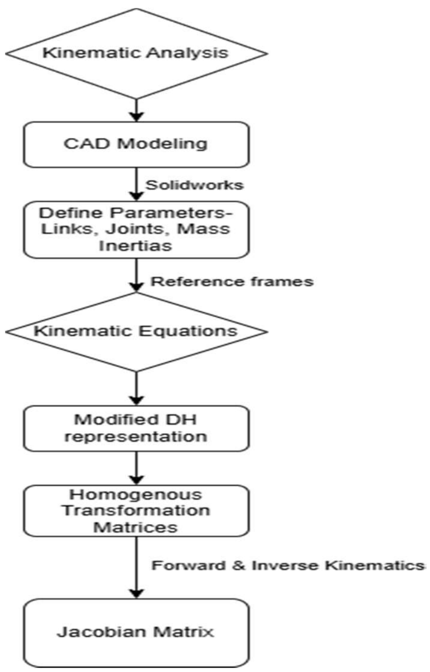
The geometric Jacobian is crucial for several robotics' applications, including trajectory generation, motion control, and solving inverse kinematics problems. By illustrating how variations in joint velocities influence the motion of the end-effector, it serves as a valuable tool for understanding and controlling robotic movements effectively.

4. Interaction Dynamics between Dual Arms and Manipulated Object

4.1 Dynamic Model of Individual arm

The dynamics of a dual-arm robotic system involve analyzing the resulting motions, forces, and torques acting on the system, including both external inputs and gravitational effects. Joint torques are computed based on known joint states—positions, velocities, and accelerations—using classical mechanics principles such as the Newton-Euler or Lagrangian methods [12]. These dynamics are critical for understanding system behavior and for developing effective control strategies under various operating conditions.

In past studies, rigid-joint robotic arms have been extensively modeled using the Newton-Euler formulation [6], [18]. This method takes into account not only joint motion profiles (q, $\dot{q}$, $\ddot{q}$) but also the forces and moments applied at the end-effector due to payload interactions. The Newton-Euler approach uses a recursive algorithm, where link velocities and accelerations are first computed from the base (link 1) up to the end-effector (link n), followed by backward propagation to determine the forces and torques required at each joint.



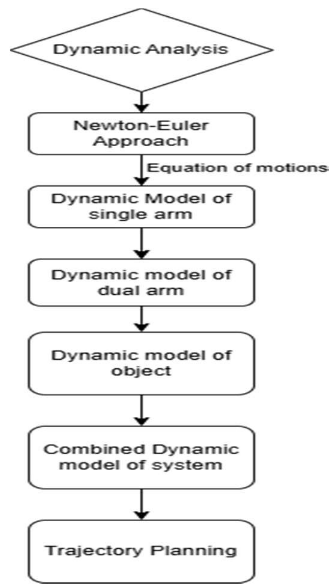


Fig. 3 Kinematic Analysis Flow Chart

Fig. 4 Dynamic Analysis Flow Char

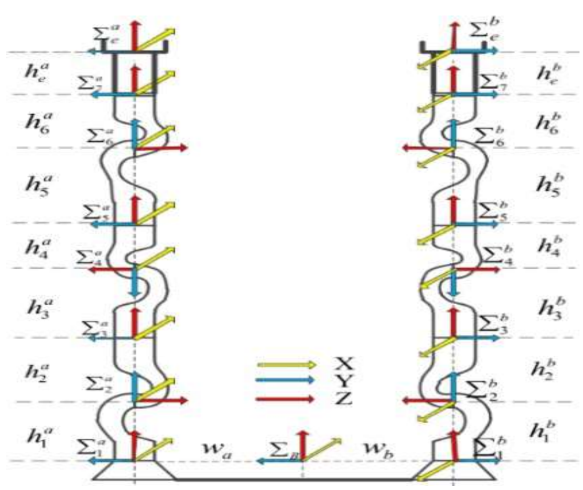


Fig. 5. The coordinate frames of Dual Arm Manipulator [6]

the backward recursion is used to compute the joint torques by applying force and moment balance equations.

$$ {}^{i}f_i = {}^{i}_{i+1}R\,{}^{i+1}f_{i+1} + {}^{i}F_i \quad (4) $$

$$ {}^{i}n_i = {}^{i}N_i + {}^{i}_{i+1}R\,{}^{i+1}n_{i+1} + {}^{i}P_{C_i} \times {}^{i}F_i + {}^{i}P_{i+1} \times {}^{i}_{i+1}R\,{}^{i+1}f_{i+1} \quad (5) $$

For each link, parameters like angular velocity (ω), angular acceleration (α), linear acceleration of the center of mass (a_cm), applied force (F), and torque (N) are calculated iteratively. The algorithm involves transforming coordinate frames and applying vector cross-product operations to compute dynamic quantities.

$$ {}^{i+1}\omega_{i+1} = {}^{i+1}R\,{}^{i}\omega_i + [0; 0; \theta_{i+1}] \quad (6)$$

$$ {}^{i+1}ac_{i+1} = {}^{i+1}_{\;i}R\,{}^{i}ac_i + {}^{i+1}_{\;i}R\,{}^{i}\omega_i \times [0; 0; \theta_{i+1}] + [0; 0; \dot{\theta}_{i+1}] \quad (7)$$

$$ {}^{i+1}lc_{i+1} = {}^{i+1}_{\;i}R\,({}^{i}ac_i \times {}^{i}P_{i+1} + {}^{i}\omega_i \times ({}^{i}\omega_i \times {}^{i}P_{i+1}) + {}^{i}lc_i \quad (8)$$

$$ {}^{i+1}lm_{i+1} = {}^{i+1}ac_{i+1} \times {}^{i+1}P_{C_{i+1}} + {}^{i+1}\omega_{i+1} \times ({}^{i+1}\omega_{i+1} \times {}^{i+1}P_{C_{i+1}}) + {}^{i+1}lc_{i+1} \quad (9)$$

$$ {}^{i+1}F_{i+1} = m_{i+1}\,{}^{i+1}lm_{i+1} \quad (10)$$

$$ {}^{i+1}N_{i+1} = {}^{C_{i+1}}I_{i+1}\,{}^{i+1}ac_{i+1} + {}^{i+1}\omega_{i+1} \times {}^{C_{i+1}}I_{i+1}\,{}^{i+1}\omega_{i+1} \quad (11)$$

The torque at each joint is obtained by projecting the computed torque vector onto the joint axis (usually the Z-axis of each frame). This recursive dynamic formulation results in an equation of the form

$$ {}_{i} = {}^{i}n^{T}\,{}^{i}Z_i \qquad {}_{i} \quad (12)$$

$$ \tau = (q)\ddot{q} + C(q, \dot{q}) + G(q) \quad (13)$$

where M(q) represents the inertia matrix, C (q, $\dot{q}$) accounts for Coriolis and centrifugal effects, and G(q) corresponds to gravitational forces [9].

4.2 Dynamic modeling of the Grasped Object in Dual-Arm Manipulation

In this section, we analyze the dynamic behavior of a rigid object grasped by a dual-arm robotic manipulator. The object is assumed to be securely held by both end-effectors (EEs), establishing a rigid connection at the grasp points, which ensures consistent motion transfer between the object and manipulators [6].

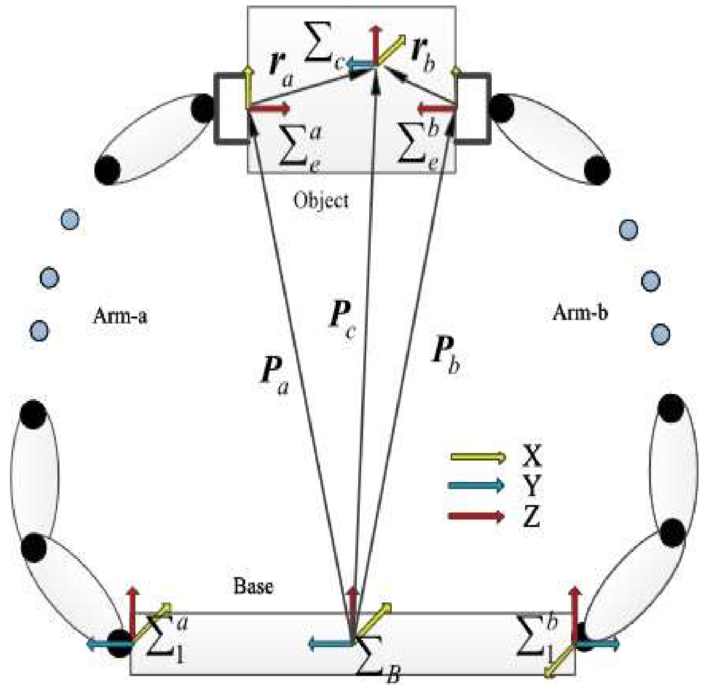


Fig. 6. The closed loop of Dual arm Robot [6]

Let the translational velocity of the end-effector be denoted as $\dot{p}_k \in R^{3\times1}$, and the angular (rotational) velocity as $\omega_k \in R^{3\times1}$, both with respect to the base frame. These can be combined into a single velocity vector: $V_k = \begin{bmatrix} \dot{p}_k \\ \omega_k \end{bmatrix} \in R^{6\times1}$

This vector relates to the object's motion through the transformation:

$$V_c^k = \begin{bmatrix} \dot{p} \\ \omega \end{bmatrix} = \begin{bmatrix} I_3 & S(r_k) \\ O_3 & I_c^B \end{bmatrix} V_c = J_{o,k} V_c \tag{14}$$

where:

- $S(r_k)$ is the skew-symmetric matrix of the position vector rkr_krk,
- I_3 is the 3×3 identity matrix,
- O_3 is a 3×3 zero matrix,
- I_B is the rotational inertia matrix of the object,
- V_C is the velocity of the object's center of mass,
- J_O, is the Jacobian matrix relating object and end-effector velocities.

Assuming rigid grasping, we derive the dynamic equations of the object using the Newton-Euler method. The general form of the object's motion is:

$$\begin{bmatrix} m_0 & O_3 \\ O_3 & I_B \end{bmatrix}\begin{bmatrix} \ddot{p}_C \\ \dot{\omega}_C \end{bmatrix} + \begin{bmatrix} -m_0 g \\ \omega_C \times I_B \omega_C \end{bmatrix} = J^T F \tag{15}$$

where:

- m_0 is the mass of the object,
- g is the gravity vector,
- ω_C is the angular velocity of the object,
- $F \in R^{6\times1}$ is the external wrench applied to the object,
- J_O is the combined Jacobian matrix of the object.

To explicitly solve for the force vector FFF, the following expression is used:

$$F = J_0^{\dagger}\left\{\begin{bmatrix} m_0 & O_3 \\ O_3 & I_B \end{bmatrix}\begin{bmatrix} \ddot{p} \\ \dot{\omega}_C \end{bmatrix} + \begin{bmatrix} -m_0 g \\ \omega_C \times I_B \omega_C \end{bmatrix}\right\} + F_I$$

Here:

- $J_0^{\dagger}$ is the pseudoinverse of J_O ,
- $F_I \in R^{6\times1}$ represents internal interaction forces distributed between the arms based on the object's handling strategy and force requirements [19, 20].

This formulation allows for comprehensive modeling of the object's response within the coupled dual- arm system, accommodating both dynamic motion and force distribution

5. Dynamic Behaviour and Torque Response of Dual-Arm System

The dynamic analysis of robotic systems is essential to understand the forces and torques required at each joint to follow a desired trajectory. In this study, the Newton-Euler Recursive Algorithm is used for inverse dynamic modeling of each 7-DOF arm of the dual-arm manipulator. This method efficiently computes joint torques by performing two main steps:

- Forward recursion: propagates angular and linear velocities and accelerations from the base to the end-effector.
- Backward recursion: calculates the forces and moments from the end-effector back to the base.

Each link of the manipulator is assigned realistic parameters, including link length, mass, and inertia tensor (randomized within a practical range). The trajectory planning is carried out using sinusoidal joint profiles, ensuring smooth variation in joint angles, velocities, and accelerations over time. Additionally, a 100 mm cube object with a mass of 2 kg is considered for cooperative manipulation by both arms. The object's dynamics are included conceptually to simulate external loading and evaluate torque requirements under realistic cooperative operation [21].

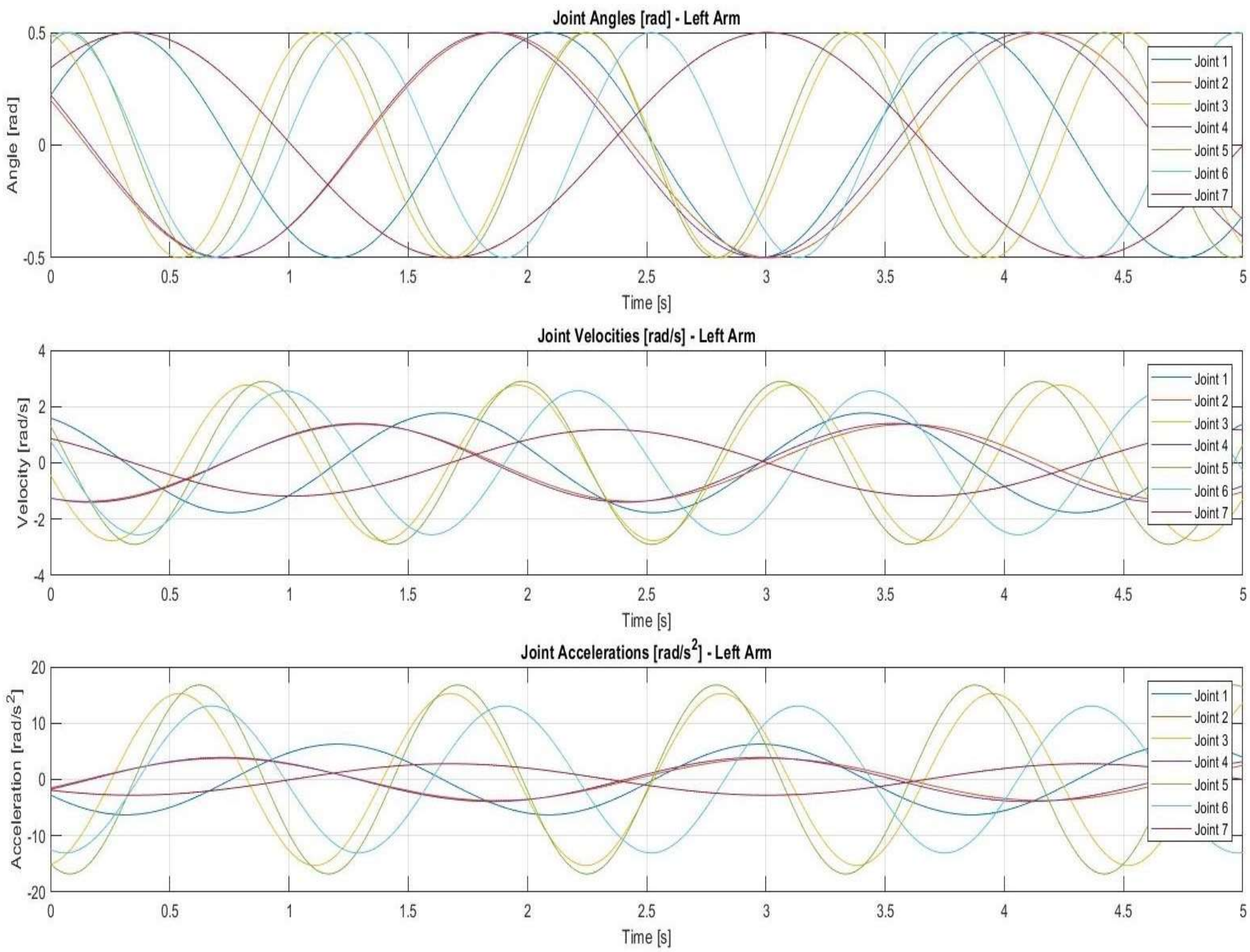


Fig. 7 Time Evolution of Joint Angles, Velocities, Accelerations, and Torques for the Left Arm

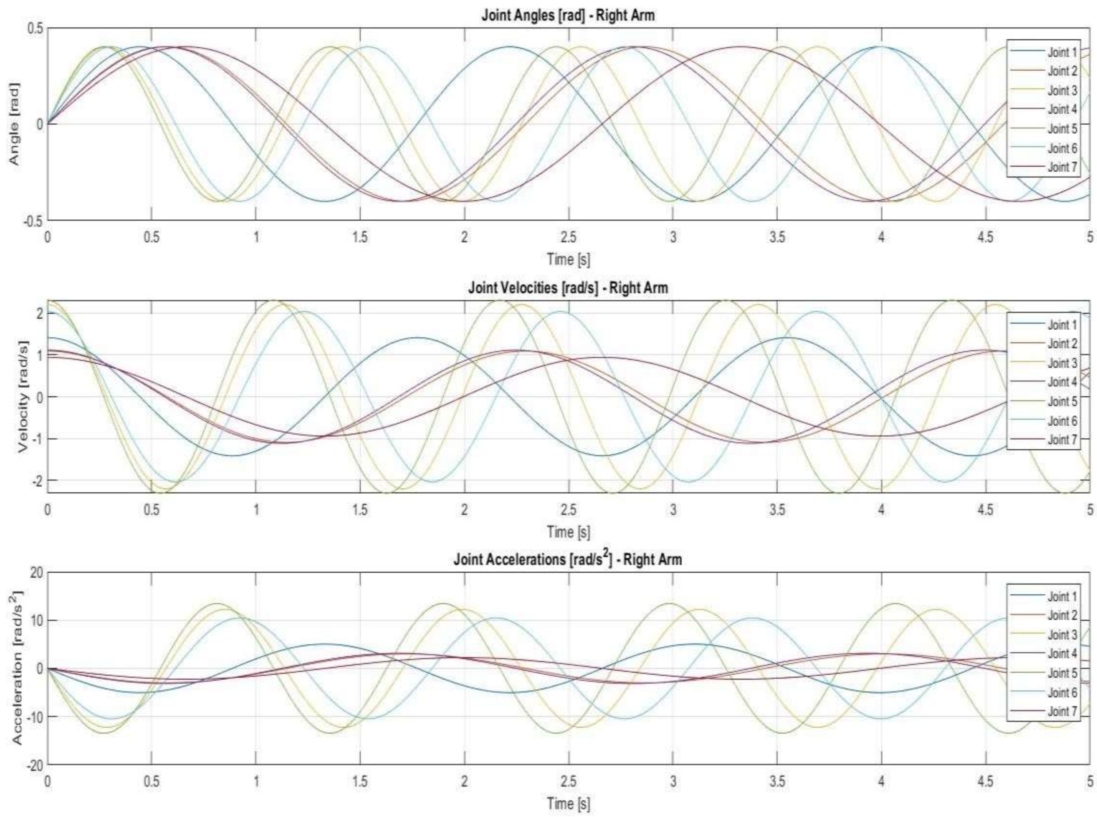


Fig. 8 Time Evolution of Joint Angles, Velocities, Accelerations, and Torques for the Right Arm

6. Future Scope

The developed dynamic modeling framework for a dual-arm cooperative robotic system opens up multiple avenues for future exploration. One significant direction involves implementing the model within a real-time control setup to validate torque predictions on a physical robot. Further, expanding the object model to include variable mass distributions or deformable properties would make the system more adaptable to practical manipulation scenarios. It can also be incorporated by using advanced grasp planning strategies and optimizing internal force-load distribution can enhance stability during cooperative tasks. Lastly, extending the system toward human-robot collaboration and multi-arm coordination in cluttered or dynamic settings holds great promise for future industrial and service applications.

7. Conclusion

The dynamic modeling of the 7-DOF dual-arm robot shows smooth joint trajectories with joint angles ranging approximately between ±0.5 radians for both the left and right arms. The corresponding joint velocities peak around ±2.5 rad/s, and the joint accelerations reach up to ±15 rad/s², indicating dynamic but stable and well-planned motion. These values ensure the joints operate within a realistic and safe range for collaborative manipulation tasks.

From the dynamic analysis using the Newton-Euler method, the required joint torques across all joints remain within the range of ±12 Nm, with the base joints (Joint 1 and 2) generally experiencing the highest torque loads due to their role in supporting the entire arm structure and object interaction. The end- effector joints (Joint 6 and 7) show lower torque demands, typically below ±3 Nm, consistent with their smaller influence on overall arm dynamics. These numerical results confirm that the trajectory planning and dynamic modeling are physically feasible, and the system is capable of performing dual-arm cooperative manipulation of a 100 mm cube (2 kg) in a dynamically stable and coordinated manner

References

1. A. Ajoudani., "Progress and prospects of the human–robot collaboration," Autonomous Robots, vol. 42, no. 5, pp. 957–975, (2018).
2. S. Niku, Introduction to Robotics: Analysis, Control, Applications, 2nd ed., Wiley, (2011).
3. R. Featherstone, Rigid Body Dynamics Algorithms, Springer, (2008).
4. S.N. Chiwande and S.S. Ohol, "Comparative Need Analysis of Industrial Robot Calibration Methodologies", IOP Conf. Series: Materials Science and Engineering 1012 012009, IOP Publishing, doi:10.1088/1757-899X/1012/1/012009, (2021)
5. C. Smith, Y. Karayiannidis, L. Nalpantidis, X. Gratal, P. Qi, D. V. Dimarogonas, and D. Kragic, ''Dual arm manipulation—A survey,'' Robot. Auto. Syst., vol. 60, no. 10, pp. 1340–1353 (2012).
6. Xin Jing, Haibo Gao, Zhengsheng Chen, And Yaobing Wang, "A Recursive Dynamic Modelling and Control for Dual-Arm Manipulator with Elastic Joints," IEEE, vol. 8, (2020).
7. S Rader, L Kaul, H. Fischbach, N. Vahrenkamp and T. Asfour, "Design of a High-Performance Humanoid Dual Arm System with Inner Shoulder Joints", IEEE-RAS 16th International Conference on Humanoid Robots, Mexico, Nov 15-16 (2016).
8. K. Benali, J. Brethe and M. Gorka, "Dual Arm Robot Manipulator for Grasping Boxes of Different Dimensions in a Logistics Warehouse", in Proc. IEEE Tnt. Conf. on Industrial Technology (ICIT), France, (2018).
9. R.S. Jamisola, P. Kormushev, D.G. Caldwell and F. Ibikunle, "Modular Relative Jacobian for Dual Arm and the Wrench Transformation Matrix", proceedings of the 2015 7th IEEE International Conference on Cybernetics and Intelligent Systems, CIS 2015 and Robotics, Automation and Mechatronics, RAM 2015, Pg. 181-186, (2015)
10. L. Yan, Z. Mu, W. Xu, and B. Yang, ''Coordinated compliance control of dual-arm robot for payload manipulation: Master-slave and shared force control,'' in Proc. IEEE/RSJ Int. Conf. Intell. Robots Syst. (IROS), Daejeon, South Korea, pp. 2697–2702 (2016).
11. M. Wang, M. Luo, T. Li, and M. Ceccarelli, ''A unified dynamic control method for a redundant dual arm robot,'' J. Bionic Eng., vol. 12, no. 3, pp. 361–371 (2015).
12. J. Liu and R. Liu, ''Dynamic modeling of dual-arm cooperating manipulators based on Udwadia–Kalaba equation,'' Adv. Mech. Eng., vol. 8, no. 7, Art. no. 168781401665884 (2016).