Crack Propagation in the Spur Gear of Tumbler Gear Mechanism of Lathe Machine

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Abstract
An estimation of the spur gear service life and crack path analysis is proposed in this paper. \( I_p \) theory using numerical solution is used to find the fatigue crack propagation rate, service life, and crack path due to bending fatigue. A spur gear of tumbler gear mechanism of lathe machine is implemented for this analysis. An experimental test on gear specimens was carried out to analyze the crack propagation due to fatigue. Besides, service life estimated by the theory was compared with the most common Paris law. Hence further, extended finite element approach (Ansys) is implemented for the simulation of gear crack propagation trajectory. The simulation results are verified with experimental test. The service life and crack trajectory of the gear is obtained with ‘\( I_p \)’ theory and found in good agreement with experiment. In overall, this study signifies the gear failure prediction methodology during crack propagation in respect of life cycles and trajectory path.

Keywords: Spur gear, fatigue crack growth, Bending fatigue, Tumbler gear mechanism; Numerical simulation

1. Introduction

The tumbler gear mechanism of the lathe machine is used to transmit the power and motion to the lead screw. The lead screw mechanism deals with automatic movement of the carriage for thread cutting, boring and turning operations, etc. The tumbler gear mechanism with gear numbers is shown in Figure 1. In this application, it has been noted that, Gear-2 of the tumbler gear mechanism frequently fails due to the consequence of crack propagation located at the gear root. This failure can be catastrophic and should be prevented. If the gear fails catastrophically, then there can be fatal accidents and financial loss to the industry or users. Prior information is necessary for service life estimation of the Gear-2 so that it could be easily replaced before the final life cycle. It is imperative to design the gears for longer life.

Figure 1: Tumbler gear mechanism of the Lathe machine

The gears are generally fails by particular reasons of the crack generation due to impulse tangential load. More care must be taken while designing the gear so that the gear can work for
specific life cycles, and the catastrophic tooth failure can be avoided. For accurate
determination of the crack propagation life, perfect understanding about crack growth and
stress intensity factor (SIF) is prerequisite.

The Gear-1 is fixed on the lathe spindle. The lead screw is connected to input shaft
through Gear train-(1-5). The Gear 2 is fitted on the motor shaft which is used for the fatigue
crack growth test because it is frequently failing. An electric motor with three p.power with a
speed of 1440 revolution per minute is used to drive the lathe spindle with chuck.

It is essential to study the fracture mechanics approach in fatigue loading for spur gear
to know the characteristics of crack and crack propagation. Analysis of the structural as well as
rotary machine component cracks can be analysed by fracture mechanics. Crack behavior in
the machine parts due to static and fatigue loading could be observed by Linear elastic fracture
mechanics (LEFM). The fracture mechanics method is generally used to determine the useful
life of the bodies applied with fatigue loading. The fatigue life of elements are divided into three
phases: 1) The phase of crack initiation, 2) The phase of crack growth, ii) the phase of the final
breakage of parts. If the crack propagation study on the component is required in a cyclically
loaded component, then fracture mechanics should be applied to fatigue propagation life
estimation.

In the past, different analytical and experimental analyses were carried out to examine the
gear crack path and crack propagation lifecycle. In some succeeding readings, authors notice
the definite effects or influences on the crack propagation. J. Kramberger and J. Flasker[1] were used in the boundary element method (BEM) and LEFM for crack propagation
analysis. Bertsche [3] investigated the damage accumulation method for high reliability and
definite gear life. Bazic et al. [4] obtained crack initiation angles by mixed-mode SIF utilizing
altered loading conditions. It was observed that the modified tangential stress (max.) criteria
were a better fit for uniaxial tensile stress in parts of a ductile material. Experimental
investigation and numerical simulation were used by Lewicki, and Ballarini[5] to studied the
thickness of the gear rim on life cycles. Numerical simulation was carried out by applying the
Fracture ANalysis Code (FRANC) computer software. The crack was fabricated with a 0.5
mm crack depth. 2D stress analysis was carried to the crack initiation CT or compact tension
specimen. Some authors analyzed the crack paths. Robert Errichello [7] used the general
AGMA and ANSI standards to analyze the crack path and crack propagation of the
gear. Glodez and Sraml [9] applied a computational model and finite element analysis using
Franc2d software to estimate the residual gear life due to cyclic bending. Glodez et al.
[10] estimated the service life of spur gears of hardened steel 42CrMo4 material due to cyclic
bending by the computational method by using experimental data was used.

Mohite and Kothavale [17] used the factorial design methodology for spur gear crack
propagation to get optimum parameters such as material, physical properties. The optimized
parameters obtained from the survey are used in the present study. Najafian and Mozafari
[18] considered the thickness of the gear rim to fix the crack propagation rate and the crack
trajectory of gear. An extended FEM and LEFM were used for the crack growth study of the
planetary gears. Osmond Asi [20] studied on the helical gear for fatigue failure analysis. The
research work presents a failure study of a helical-gear type realized in the transmission
gearbox of a bus. Paris and Erdogan [22] analyzed different crack propagation laws, and the
new law was investigated. The Paris law was employed to determine the crack propagation rate
and service life cycles of the components. Penhan et al. [23] studied crack propagation in the
tooth root portion. A particular instrument was used for testing the crack initiation and
growth. It concludes that the tooth is frequently failed due to the defect like mechanical
treatment undercut, tracks. Ukadgaonkar [25] used $I_p$ theory in CT specimens to decide the fatigue crack propagation rate. This rate is having a functional relationship with the difference between the maximum and minimum $I_p$ parameters. Ukadgaonkar and Awasare [26] found that distortion strain energy remained stable in the critical radius circle around crack tip. The crack was propagated along, determined the parameter $I_p$. It can give the magnitude and the orientation of the crack. Lu Xi [28] applied the detailed notes on the gear in the crack initiation area to find remaining stress, hardness. The crack of 0.5 mm had been fabricated at the root of the gear.

In this analysis work, an effort is made to estimate the useful service lifecycle of the gear and fix the crack trajectory, specifically, implemented in the mechanism of the tumbler gear of the lathe machine. A test on gear carried out to analyze the crack propagation; the results were compared with the Paris law. Numerical simulation was performed to decide the SIF and model crack propagation. FEA (Ansys) was applied to analyze crack growth. Therefore, the study aims to find the remaining useful lifespan of the tumbler gear to escape from a catastrophic failure.

2. Material and method

2.1. Material

A spur gear test specimen was used for the analytical and experimental studies. The spur gear material is alloy steel (SAE8620). The chemical composition, physical properties, and geometrical parameters of the material are shown in Tables 1, 2, and 3, respectively. The optimized gear parameters and the material were determined by the same Authors based on the factorial design approach in a previous study [2]. The same material, geometric parameters, and mechanical properties are considered in the current work, as shown in following Table 1, and Table 2.

<table>
<thead>
<tr>
<th>Material</th>
<th>Young's modulus, $E$ (MPa)</th>
<th>Yield tensile strength, $S_y$ (MPa)</th>
<th>Ultimate tensile strength, $S_{ut}$ (MPa)</th>
<th>Fracture toughness, $K_{ic}$ (MPa√m)</th>
<th>Poisson's ratio, $ν$</th>
<th>Initial Crack length, $a_i$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAE 8620</td>
<td>$2 \times 10^5$</td>
<td>390</td>
<td>540</td>
<td>75</td>
<td>0.29</td>
<td>0.125</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number of Teeth, $Z$</th>
<th>Module, $m$, (mm)</th>
<th>Pitch circle diameter, $d$, (mm)</th>
<th>Pressure angle, $Ø$, (degrees)</th>
<th>Face width, $b$, (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>28</td>
<td>2.5</td>
<td>390</td>
<td>$20^0$</td>
<td>12.5</td>
</tr>
</tbody>
</table>
2.2. Theoretical treatments

2.2.1. The Ip theory

Ukadgaonkar and Awasare [25] investigated Ip theory for fatigue fracture, a new crack initiation criterion which was based on dilatational strain energy. It is useful to identify the crack initiation angle of the machine components applied with the dynamic or static load. The present study is used

➢ To decide the crack growth rate and estimation of gear life under fatigue loading.
➢ To determine the crack trajectory in the tooth of the gear under static or fatigue loading.

State of stress generated on tip of elastic body crack has been studied by the Irwin. Further, derived the expressions for \( \sigma_x \) and \( \sigma_y \) concerning crack propagation angle \( \theta \). The 2-D stresses at the crack tip are \( \sigma_x \) and \( \sigma_y \) in X and Y directions, respectively [21].

Ip theory is depending upon two stress invariants \( I_1 \) and \( I_2 \). \( I_1 \) is the first stress invariant, and \( I_2 \) is other stress invariant. The relation between the stresses at the crack tip and stress invariants \( (I_1) \) and \( (I_2) \) are given in Eq. (1) & Eq. (2) respectively.

The stress Invariant, \( I_1 \),
\[
I_1 = \sigma_x + \sigma_y \tag{1}
\]

The stress Invariant, \( I_2 \),
\[
I_2 = \sigma_x \sigma_y - \tau_{xy}^2 \tag{2}
\]

Ip is the parameter, on which Ip theory was postulated by Eq.(3),
\[
I_p = I_1^2 - 2I_2 = 1/2\pi r \left( c_{11}K_i^2 + c_{22}K_{II}^2 + c_{12}K_iK_{II} \right) \tag{3}
\]

Where,
\[
c_{11} = 1/2 \left( 3 + 2 \cos \theta - \cos^2 \theta \right), c_{22} = 1/2 \left( 3 - 2 \cos \theta + 3 \cos^2 \theta \right) and c_{12} = 2 \sin \theta (\cos \theta - 1)
\]

The terms \( K_{I} \) and \( K_{II} \) are the SIF’s and ‘\( \theta \)’ is an angle of crack propagation.
\[
I_p = \frac{1}{2\pi r} \left[ \left( \frac{1}{2} \left( 3 + 2 \cos \theta \cos^2 \theta \right) K_i^2 + \frac{1}{2} \left( 3 - 2 \cos \theta + 3 \cos^2 \theta \right) K_{II}^2 + 2 \sin \theta (\cos \theta - 1) K_i K_{II} \right) \right] \tag{4}
\]

Differentiating and double differentiating Eq.(4)
\[
\frac{\partial I_p}{\partial \theta} = 0 \text{ and } \left( \frac{\partial^2 I_p}{\partial^2 \theta} \right) < 0 \tag{5}
\]

The maximum value of the \( \theta \) is determined by Eq. (5). As per Ip theory, the crack can be advanced in the way of the extreme value of parameter Ip. Ip parameter is directly proportional to the dilatational strain energy density. The dilatational strain energy is the main cause of crack propagation.

For the maximum value \( I_p \), the differentiation of the Eq. (5) in with respect to angle ‘\( \theta \)’, is zero.
\[
\frac{\partial I_p}{\partial \theta} = 0 = \left[ (-\sin \theta + \frac{1}{2} \sin \theta) K_i^2 + (\sin \theta - \frac{3}{2} \sin 2\theta) K_{II}^2 + (2 \cos 2\theta - 2 \cos \theta) K_i K_{II} \right] \tag{6}
\]

Rearranging the Eq. (6),
\[
\left[ \left( K_i^2 - 3K_{II}^2 \right)/2 \right] \sin 2\theta + 2K_i K_{II} \cos 2\theta - \left( K_i^2 - K_{II}^2 \right) \sin \theta - 2K_i K_{II} \cos \theta = 0 \tag{7}
\]

Crack propagation angle \( \theta \) can be determined from ‘Eq.(7)’.

Differentiating Eq.(7) with respect to \( \theta \),we get,
\[ \frac{\partial^2 I_p}{\partial \theta^2} < 0 \]  

Eq. (8) must be satisfied as per guidelines of Ip theory.

Ip theory of fracture mechanics proposed by Ukadgaonkar and Awasare [25] and used for fatigue loading by Ukadgaonkar [26]. Further, it is used to calculate the crack growth life. This theory can be applied for the machine components subjected to mode-I fatigue loading. It has been proposed that the crack propagation level was in a logarithmic relation to the range of the stress invariant \( \Delta I_p \) and given as:

\[ \frac{da}{dN} = C(\Delta I_p)^m \]  

Fatigue crack growth rate using Ip theory is prescribed in Eq. (9). Where \( dN \) is the number of stress cycles and \( da \) length of crack correspond to \( dN \). \( \Delta I_p \) is the difference of the stress invariants which can be determined from Eq. (11), (12) and (13)

\[ \Delta I_p = C \left( \frac{\Delta I_p}{C} \right)^m \]  

\[ R = \int_{\alpha}^{\beta} \frac{da}{C(\Delta I_p)^m} \]  

The \( \Delta I_p \) is a change in Ip max and Ip min, the material parameters \( C \) and \( m \) can be determined from the experiment of three point bending test on single edge notched beam using standard procedure given in ASTM E 399-80.

For fatigue loading, there are maximum and minimum stresses, namely, ‘\( \sigma_{\text{max}} \)’ and ‘\( \sigma_{\text{min}} \)’ respectively.

Stress intensity factors are prescribed in Eq. (11),

\[ K_{\text{max}} = 1.12 \sigma_{\text{max}} \sqrt{\pi a} \left( \text{MPa}\sqrt{\text{mm}} \right) \text{and} \ K_{\text{min}} = 1.12 \sigma_{\text{min}} \left( \text{MPa}\sqrt{\text{mm}} \right) \]  

Stress invariant parameters of Ip can be evaluated by using the Eq. (12),

\[ I_{p_{\text{max}}} = (K_{\text{max}})^2 / (\pi da) \text{and} \ I_{p_{\text{min}}} = (K_{\text{min}})^2 / (\pi da) \]  

The range of the stress invariants \( \Delta I_p \) is given by,

\[ \Delta I_p = (I_{p_{\text{max}}} - I_{p_{\text{min}}}) \left( \text{MPa}\sqrt{\text{mm}} \right)^2 / \text{mm} \]  

2.1.2. Bending fatigue stress

The gear tooth is here, presumed to be a cantilever beam. The length, width, and depth of the cantilever beam are gear tooth depth, face width, and tooth thickness respectively. The gear tooth is applied with a tangential tooth load. The tangential tooth load \( P_t \) was calculated from the power and velocity of the electric motor, used in the experimental set-up. The calculated tooth load \( P_{t\text{s}} \) is 423.93 N. The basic rack tooth profile type C is suggested in (ISO 53) standard [11] for normal working conditions. Type C profile was used for the manufacturing of the spur gear with hob cutter.

The dimensions of critical cross section at the root gear tooth, which was determined as per the procedure of ISO 6336-6 standard [12] The tooth dimensions at critical cross section were found from the functional relations given in the standard. \( S_{\text{fn}} \) is 5.059 mm and face width, \( b \) is 12.5 mm. Tooth depth from HPSTC to critical section, \( h_{\text{fn}} \) is 3.58 mm. Putting these values in Eqns. (14), (15) and (16), we get:

Bending moment (BM) at the root area of the tooth,

\[ M = P_t h_{\text{fn}} = 1517 \text{ N}\cdot\text{mm} \]  

103
Section Modulus at the critical section;

\[ Z = \frac{b s_n^2}{6} = 53.336 \text{ mm}^3 \]  

(15)

Bending Stress \( (\sigma_b) \) at the area of tooth root is given as;

\[ \sigma_b = \frac{M}{Z} = 28.44 \text{ N/mm}^2 \]  

(16)

This bending stress is in cyclic nature because tangential impulse load acting on gear.

2.3. Numerical simulation

2.3.1. Modelling

The SOLIDWORKS is computer-aided design (CAD) solid modelling software. It runs mainly on Microsoft Windows. The design engineers can sketch out their ideas, creates models and detailed drawings with dimensions. It can generate 2D and 3D models. Spur gear solid model was created in Solid-works software. The geometric parameters of the spur gear were referred from Table 2.

The initial crack length was assumed as 0.125mm (Glodez et al., 2002). The crack, having depth 0.125 mm and width 0.13, was performed at tooth root. The crack was inclined at 45° parallel to tooth axis (Lewicki, and Ballarini, 1997). The magnified view of the gear tooth root as shown in Figure 2 (a). Figure 2(b) shows the gear tooth with a crack. The pre-meshed crack model of the gear is shown in Figure 3.

Figure 2: (a) Magnified view of the gear tooth root with crack dimension, (b) Gear tooth with a crack

Figure 3: Pre-meshed crack in 3D

Figure 4: Meshed Model in 3D
2.3.2. Analysis of crack propagation

Ansys software is implemented to analyse the crack propagation of cracked gear. Its finite element software for static and dynamic analysis for two-dimensional and three-dimensional problems of the plane strain and plane strain. The gear model was imported in Ansys software from SOLIDWORKS. Ansys software generated the meshing in the gear model. A meshed model of the gear sector is shown in Figure 4. The geometric and material constraints of the gear are similar to the test gear, used in the experimental setup. The required geometrical and material properties of SAE 8620 were assigned to the gear model. The physical and geometrical properties were referred from Tables 1, 3. The model is having the six-node tetrahedron element. The meshed model had 62936 nodes and 10364 six-node tetrahedron elements. Ansys had calculated stress intensity factor (SIF), crack increment and the crack propagation angle. SIF was calculated for the crack increments associated with the number of cycles.

The Paris law in Eq. (14) was used in numerical simulation to estimate the crack propagation rate. Next, the service life is estimated by the functional relation of crack propagation rate and SIF. Paris law constants C and m were also assigned. The values C is 2.2 x 10^-12 mm/(cycle.MPa.mm^{0.5}) and m is 2.7. The tangential tooth load was assumed to be acting at HPSTC. The effect of radial component is not considered as it is assumed to be negligible. For the required boundary condition, the left sides of the gear sector and hub nodes were fixed as shown in Fig 5.

![Figure 5: Boundary condition](image)

The boundary condition in which, the tangential force of 423.91 N pulsating in nature, is applied at the HPSTC, as shown in Figure 5. The fixed supports are applied on the left side and at the shaft hole position of the gear sector.

3. Results and Discussion

3.1. Numerical simulation

Numerical simulation was carried out for determination of stress intensity parameter, fatigue crack propagation life and crack trajectory. The terms are explained and determined below.

3.1.1. Mode-I Stress Intensity factor

The most dominating factor in crack propagation analysis is the mode-I SIF (K_i) which is required to determine the service life as well as crack propagation direction. It can also establish functional relationship between \( \Delta K = f(a) \).
The functional relation between the Mode-I SIF and crack extension $a$, where $K$ is determined numerically. Figure 6 shows a graph of the no. of load cycles and crack extension, which was found from the simulation of crack propagation on gear. The numerical value of $K_I$ is higher than $K_{II}$ ($K_I$ is 95% of $K$, so $K_{II}$ is ignored from calculation), so the Mode-I fracture toughness, $K_{IC}$ is considered as critical parameter. The critical crack length can be determined from $K_{IC}$ with applied stress of 28.44 N/mm$^2$.

3.1.2. Service life estimation by Numerical simulation

Numerical simulation was performed using Ansys software as explained in section 2.2. The numbers of cycles with respect to crack length were determined and it is shown in Figure 7. In observation, the rise in crack length is very less up to mid-point of the curve, and crack length increases rapidly in further cycles.

The numerical simulation is performed up to the crack length of 3.3759 mm for number $3.4261 \times 10^7$ cycles. But the final crack length just before gear tooth failure is assumed to be 4.2 mm. So, it is necessary to find the no. of cycles up to crack length of 4.2 mm (i.e. final crack length).
The crack lengths (next to 3.3759 to 4.20 mm) got from numerical simulation correspond to no. of cycles are obtained by $I_p$ theory Eq.(10) are shown in Table 3 and used in Graph shown in Figure 7.

<table>
<thead>
<tr>
<th>Crack length</th>
<th>3.65</th>
<th>3.74</th>
<th>3.83</th>
<th>3.92</th>
<th>4.01</th>
<th>4.12</th>
<th>4.20</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of cycles</td>
<td>34278000</td>
<td>34295000</td>
<td>34312000</td>
<td>34329000</td>
<td>34346000</td>
<td>34363000</td>
<td>34380000</td>
</tr>
</tbody>
</table>

3.1.3. The useful service life of the gear using $I_p$ Theory

It is required to find the numbers of cycles from initial to final crack length by Eqs. (10). The crack length just before the tooth failure in experimental method is 4.2 mm. The initial length of crack, $a_i$ is 0.125 mm and final crack length, $a_f$ is 4.2 which was determined from the experiment. The parameters $C$ is $7.002 \times 10^{-9}$ mm/cycle (MPa)$^2$ and $m=0.5678$ The values were obtained from the fatigue crack growth experiment on single edged notched specimen (SENB). The $C$ and $m$ parameters are determined similarly to Paris law using SENB specimen. Final number of cycles,

$$N = \int_{a_i=0.125}^{a_f=4.2} \frac{da}{\left(7.002 \times 10^{-9} \left(1.12 \times 28.44 \sqrt{\pi a} \right)^{0.5678}\right)}$$

$$= 3.5145 \times 10^7 \text{cycles}$$

Hence, the useful service life of the gear is $3.5145 \times 10^7$ cycles up to final crack length of 4.2 mm.

The final crack length using numerical simulation was 3.3759 mm only. But the final crack length was assumed to be 4.2mm. The remaining crack lengths corresponds to no. of cycles are obtained by Eq.(10) of $I_p$ theory and used in Graph shown in Figure 7.

The service life cycle of the gear by simulation method with $I_p$ theory is $3.5145 \times 10^7$ cycles.

3.2. Determination of crack trajectory using numerical method

3.2.1. Crack trajectory using numerical simulation with Ip theory

For identification of the crack trajectory path in mixed modes-I and II, a spur gear with a preliminary crack under tangential tooth load $P_t$ was used as shown in Figure 5. The crack path data obtained from numerical simulation was used for determination of crack trajectory. The numerical simulation with the ‘$I_p$’ theory was used to find crack direction in the spur gear tooth. Figure 8 shows the crack trajectory at the gear tooth.

SIF at crack-tip of gear tooth

$$K = \sigma_{max} \sqrt{\pi a}$$  \hspace{1cm} (17)

Mode-I SIF,

$$K_I = K \sin^2 \theta$$  \hspace{1cm} (18)

Mode-II SIF,

$$K_{II} = K \sin(\theta) \cdot \cos(\theta)$$  \hspace{1cm} (19)
Eqs.(1) to (8) give an angle $\theta$ at every crack advancement. Thus, angles are found out by the $I_p$ theory for the observations and the graph was plotted. Using crack data, the path of the crack was determined shown in Fig.15. The coordinates $x$ and $y$ of the crack tip were considered for finding the crack path crack trajectory. The fatigue crack propagation angles were determined from Eqs. (1) to (8).

The procedure to determine the crack trajectory: 1) First crack length with respect to no of cycles was obtained from numerical simulation. 2) Stress Intensity factor was determined 3) Initial the crack length of 0.125 mm and angle of crack of 45°were considered. For the next crack length was assumed to be inclined at 0°. The respective stress intensity factor $K$, the mode-I and mode-II SIF are calculated from the Eqs.(18) and (19). Putting the values in Eq.(7), the crack propagation angle $\theta$ was obtained for every crack advancement. The procedure is repeated for remaining cycles. Thus crack trajectory is determined. Figure 9 shows the output of Ansys software i.e. the crack trajectory at the root of the gear. The crack trajectory is in concave nature and along the tooth thickness.

![Figure 8: The crack trajectory at the root of the gear](image)

It is observed from Figure 8 and Figure 9 that the crack trajectory is curved and it is propagated from crack initiation point towards the opposite side of the tooth root.

4. Conclusion

Numerical simulation conducted for the service life assessment and the crack trajectory at the tooth root, due to bending fatigue. The gear of tumbler gear mechanism is considered for this study to predict the stage before the gear tooth failure.

- $I_p$ theory has been successfully implemented to estimate the service life of the spur gear. Estimated life cycles by $I_p$ theory are $3.5145 \times 10^7$.
- The crack trajectories of the gear specimen in FEA and FEA with $I_p$ theory are found in close match. The curved crack path profiles are keenly observed and found that, it propagates towards the opposite side of the gear tooth along the face width.

In the present work, the simulation gives the perfect idea about prediction of the spur gear service life and its early maintenance or replacement before the failure.

References


